

Corrigendum to “A note on some Diophantine inequalities over adelic curves”

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Abstract

We correct a reversed inequality on the parameter A in the statements of [1, Theorem 1.2 and Proposition 3.9], and take the opportunity to refine the statement of Proposition 3.9 and to streamline the proof of Theorem 1.2. We also replace system (1.1) by a system with prescribed targets on a fixed measurable partition. A few minor typos are also listed. The cardinality bound in Theorem 1.2 is unchanged.

The approximation system. On p. 144 of [1], fix a measurable partition $S = S_1 \sqcup \cdots \sqcup S_n$ (empty parts allowed), require $\theta \in L^1(S, \mu)$, and replace $\omega \in S$ by $\omega \in S_i$ in (1.1). The partition and θ are independent of β . In (1.2), take $\alpha_\omega = \alpha_i$ on S_i , assuming $\beta \neq \alpha_i$ for all i and $\log A + h(\beta) > 0$. Under the corrected hypothesis on A below, the system as printed has no solutions with $h(\beta) \geq \log A$, by Proposition 2.7 applied to a target of minimum height. Numbering below refers to the published paper.

1. The hypothesis on A . The inequality on A in the statements of Theorem 1.2 and Proposition 3.9 of [1] is reversed. The correct version reads

$$A \geq 4^{2N!} \prod_{i=1}^n H(\alpha_i)^{2N!/n}$$

in both places. Under this corrected hypothesis on A :

- (a) Proposition 3.1 should be restated under the additional assumption $A \geq 4^{2N!} \prod_{i=1}^n H(\alpha_i)^{2N!/n}$; with this assumption, hypothesis (3.1) can be replaced by $h(\beta_2) \geq h(\beta_1) > 0$. Indeed, the proof of Proposition 3.1 derives

$$\frac{h(\beta_2)}{h(\beta_1)} \geq 1 + \varepsilon + \frac{(2 + \varepsilon) \log A - h(2) - \log 2}{h(\beta_1)},$$

and the corrected hypothesis implies $\log A \geq 4 \log 2$, which (with $h(2) \leq \log 2$) makes the fractional term strictly positive.

- (b) The height threshold in Theorem 1.2 simplifies to $h(\beta) \geq \log A$ (by (a) the lower bound on $h(\beta_1)$ in (3.1) is no longer required).
- (c) The clause beginning “*moreover when $A \leq 1 \dots$* ” in the paragraph following Theorem 1.2, together with the displayed inequality and the trailing words it introduces, should be deleted, and the preceding semicolon replaced by a full stop.
- (d) Remark 3.2 should be removed.

2. Proposition 3.9 and the proof of Theorem 1.2. In Proposition 3.9, replace “*logarithmic length less or equal than Γ* ” by “*logarithmic length less or equal than $\log \Gamma$* ”, where $\Gamma = 8nN^2N!$; the intervals contain the *heights* of the solutions. Before applying Theorem 3.8, note that $N! \geq n$

gives $\log A > \max_i h(\alpha_i)$, hence $\beta_j \neq \alpha_i$. Moreover, θ is column bounding on the fixed partition because $\log \rho_j \leq h(\beta_j) + \log A$.

Proposition 3.1 makes the height injective on the solutions under consideration and their heights locally finite. Starting at the least uncovered height $a \geq \log A > 0$, choose $[a, \Gamma a]$. This requires at most $N - 1$ intervals, since N left endpoints would give the N -tuple excluded in the proof of Proposition 3.9.

For Theorem 1.2, let $x_1 < \cdots < x_r$ be the heights in one such interval. By 1(a), for $r \geq 2$,

$$(r - 1) \log(1 + \varepsilon/2) < \log(x_r/x_1) \leq \log \Gamma.$$

Thus $r < 1 + \log \Gamma / \log(1 + \varepsilon/2)$, also for $r = 0, 1$. Summing over at most $N - 1$ intervals gives the stated cardinality bound.

3. Typos and clarifications.

1. In Definition 2.6, the integration range S should read Ω .
2. In Definition 2.8, the triple $(\mathbb{K}, \Omega, \mu)$ should read $(\mathbb{K}, \Omega, \phi)$, and the bounded-height set should read $\{\alpha \in \mathbb{K} : h(\alpha) \leq C\}$, with $h(0) = 0$.
3. In the proof of Proposition 3.1, the integration ranges Ω_∞ , Ω_0 , Ω should be replaced by $S \cap \Omega_\infty$, $S \cap \Omega_0$, S respectively, and Proposition 2.7 should be applied with $U = S$; at $\omega \in S_i$, use α_i in both terms of the triangle inequality. The conclusion is unchanged.
4. In Theorem 3.8, the matrix $T(\beta)$ belongs to $M((n + 1) \times N, \mathbb{K})$.
5. In equation (3.7), $\sum_{i=1}^n \frac{N!}{n} h(\alpha_i)$ in the denominator should read $\sum_{i=1}^n \frac{N!}{2^n} h(\alpha_i)$, in agreement with Definition 3.5.
6. In Proposition 2.7, assume $\alpha \neq \beta$.
7. In Definition 3.3, delete the sentence about defining logarithmic length by translation; the definition $\log(b/a)$ applies for $0 < a \leq b$, with length zero for a singleton.
8. In the column-bounding display following (3.7), β should read β_j and the range is $\omega \in S_i$. At the end of that proof, $\int_S \theta d\mu > 2 + \varepsilon$ should read $\int_S \theta d\mu \geq 2 + \varepsilon$; the contradiction is with the strict upper bound in Theorem 3.8.

References

- [1] P. Dolce, *A note on some Diophantine inequalities over adelic curves*, J. Théor. Nombres Bordeaux **37** (2025), no. 1, 143–152.