# Corrigendum to "A note on some Diophantine inequalities over adelic curves"

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**Abstract**

We correct a reversed inequality on the parameter $A$ in the statements of [1, Theorem 1.2 and Proposition 3.9], and take the opportunity to refine the statement of Proposition 3.9 and to streamline the proof of Theorem 1.2. A few minor typos are also listed. The cardinality bound in Theorem 1.2 is unchanged.

**1. The hypothesis on $A$.** The inequality on $A$ in the statements of Theorem 1.2 and Proposition 3.9 of [1] is reversed. The correct version reads

$$A \;\geq\; 4^{2N!}\prod_{i=1}^{n} H(\alpha_i)^{2N!/n}$$

in both places. Under this corrected hypothesis on $A$:

(a) Proposition 3.1 should be restated under the additional assumption $A \geq 4^{2N!}\prod_{i=1}^{n} H(\alpha_i)^{2N!/n}$; with this assumption, hypothesis (4) can be replaced by $h(\beta_2) \geq h(\beta_1) > 0$. Indeed, the proof of Proposition 3.1 derives

$$\frac{h(\beta_2)}{h(\beta_1)} \;\geq\; 1+\varepsilon+\frac{(2+\varepsilon)\log A - h(2) - \log 2}{h(\beta_1)},$$

and the corrected hypothesis implies $\log A \geq 4\log 2$, which (with $h(2) \leq \log 2$) makes the fractional term non-negative.

(b) The height threshold in Theorem 1.2 simplifies to $h(\beta) \geq \log A$ (by (a) the lower bound on $h(\beta_1)$ in (4) is no longer required).

(c) The clause beginning "*moreover when* $A \leq 1\ldots$" in the paragraph following Theorem 1.2, together with the displayed inequality and the trailing words it introduces, should be deleted, and the preceding semicolon replaced by a full stop.

(d) Remark 3.2 should be removed.

**2. Proposition 3.9 and the proof of Theorem 1.2.** In the statement of Proposition 3.9, the phrase "*logarithmic length less or equal than* $\Gamma$" should be replaced by "*logarithmic length less or equal than* $\log\Gamma$", where $\Gamma = 8nN^2N!$ and the logarithmic length of an interval $[a,b]$ is $\log(b/a)$ (Definition 3.3); the proof of Proposition 3.9 indeed establishes this. The proof of Theorem 1.2 should then be replaced by the following direct argument. Let $x_1 < \cdots < x_r$ be the heights in one such interval, so that $\log x_r - \log x_1 \leq \log\Gamma$. By Proposition 3.1, $\log x_{j+1} - \log x_j \geq \log(1+\varepsilon/2)$ for each $j$, hence $(r-1)\log(1+\varepsilon/2) \leq \log\Gamma$ and $r \leq 1 + \log\Gamma/\log(1+\varepsilon/2)$. Summing over the at most $N-1$ intervals gives the cardinality bound of Theorem 1.2 unchanged.

**3. Typos.**

1. In Definition 2.6, the integration range $S$ should read $\Omega$.

2. In Definition 2.8, the triple $(\mathbb{K}, \Omega, \mu)$ should read $(\mathbb{K}, \Omega, \phi)$.

3. In the proof of Proposition 3.1, the integration ranges $\Omega_\infty$, $\Omega_0$, $\Omega$ should be replaced by $S \cap \Omega_\infty$, $S \cap \Omega_0$, $S$ respectively, and Proposition 2.7 should be applied with $U = S$; the conclusion is unchanged.

4. In Theorem 3.8, the matrix $T(\boldsymbol{\beta})$ belongs to $M((n+1) \times N, \mathbb{K})$.

5. In equation (10), $\sum_{i=1}^{n} \frac{N!}{n} h(\alpha_i)$ in the denominator should read $\sum_{i=1}^{n} \frac{N!}{2n} h(\alpha_i)$, in agreement with Definition 3.5.

## References

[1] P. Dolce, *A note on some Diophantine inequalities over adelic curves*, J. Théor. Nombres Bordeaux **37** (2025), no. 1, 143–152.