Esercizio 8.27

Risolvere le seguenti disequazioni goniometriche:

1. $4\sin^2 x - \sqrt{3}\sin x\cos x + \cos^2 x \leq 1$;
2. $\sin x + \cos x \geq \sqrt{2}$;
3. $1 + \cos x < 2\cos^2 x$;
4. $\cos^2 x - |\sin x| > 1 + \sin x$;
5. $(1 + 2\,|\cos x|)\,(1 - \sin x) > 0$;
6. $\left(2\cos^2 x - 1\right)(2\sin x + 3) \geq 0$;
7. $\dfrac{\tan x}{\cos x - 1} + \sin x < 0$;
8. $\dfrac{\tan^2 x - 1}{\cos(2x)} > 1$;
9. $\sqrt{5 - 2\sin x} \geq 6\sin x - 1$;
10. $\dfrac{|\sin x|}{1 - \sin x} < 1$;
11. $\dfrac{\sqrt{3}\cos x + \sin x}{\sqrt{3}\sin x + \cos x} \geq 0$;

12. $\dfrac{2}{1+\tan^2 x} - 3\sin x > 0$;

13. $\log_{\frac{1}{2}}\left(\dfrac{1}{2} - |\sin x|\right) > 0$;

14. $\cos\left(\log_3(2x)\right) < 1$;

15. $\dfrac{x^4 - x^2 - 6}{\log_{10}^2\left(\sin x - \dfrac{1}{2}\right)} \geq 0.$

Risposta:

1. $\left[k\pi, \dfrac{\pi}{6} + k\pi\right], \quad k \in Z$;

2. $x = \dfrac{3}{4}\pi + 2k\pi, \quad k \in Z$;

3. $\left]\dfrac{2}{3}\pi + 2k\pi, \dfrac{4}{3}\pi + 2k\pi\right[, \quad k \in Z$;

4. $\emptyset$;

5. $x \neq \dfrac{\pi}{2} + 2k\pi, \quad k \in Z$;

6. $\left[-\dfrac{\pi}{4} + k\pi, \quad \dfrac{\pi}{4} + k\pi\right], \quad k \in Z$;

7. $\left]2k\pi, \quad \dfrac{\pi}{2} + 2k\pi\right[\cup \left](2k+1)\pi, \quad \dfrac{3}{2}\pi + 2k\pi\right[, \quad k \in Z$;

8. $\emptyset$;

9. $\left[-\dfrac{7}{6}\pi + 2k\pi, \dfrac{\pi}{6} + 2k\pi\right], \quad k \in Z$;

10. $\left]-\dfrac{7}{6}\pi + 2k\pi, \dfrac{\pi}{6} + 2k\pi\right[, \quad k \in Z$;

11. $\left]-\dfrac{\pi}{6} + k\pi, \dfrac{2}{3}\pi + k\pi\right], \quad k \in Z$;

12. $\left]-\dfrac{7}{6}\pi + 2k\pi, \dfrac{\pi}{6} + 2k\pi\right[, \quad k \in Z$;

13. $\left]-\dfrac{\pi}{6} + k\pi, \dfrac{\pi}{6} + k\pi\right], \quad k \in Z$;

14. $x \neq \dfrac{3^{2k\pi}}{2}$;

15. $\sqrt{3} \leq x < \dfrac{5}{6}\pi.$