Sheet 6: Computations

June 10, 2024

1. Calculate the following integrals in $\mathbb{R}^3$:

(a)

$$\iiint_D dxdydz \; ; \quad D = \{(x,y,z) \in \mathbb{R}^3 \colon x^2 + y^2 \leq ze^{z^2-1}\,, 0 \leq z \leq 1\}$$

(b)

$$\iiint_D (x^2 + z)dxdydz \; ; \quad D = \{(x,y,z) \in \mathbb{R}^3 \colon x^2 + y^2 \leq z \leq 4\}$$

(c)

$$\iiint_D ze^{x^2+y^2} dxdydz \; ; \quad D = \{(x,y,z) \in \mathbb{R}^3 \colon x^2 + y^2 \leq z \leq 1\}$$

(d)

$$\iiint_D \left(\sqrt{x^2+y^2} - z\right) dxdydz \; ; \quad D = \{(x,y,z) \in \mathbb{R}^3 \colon x^2+y^2+z^2 \leq z \leq 1\,, x^2+y^2 \leq z^2\}$$

2. Study the critical points of the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ restricted to the constraint S in the following cases:

(a)

$$f(x,y) = x^2y - 2 \; ; \quad S = \{(x,y) \in \mathbb{R}^2 \colon y - x^2 + 2x = 0\}$$

(b)

$$f(x,y) = e^{xy} + xy \; ; \quad S = \{(x,y) \in \mathbb{R}^2 \colon x^2 + 2y^2 \leq 1\}$$

3. Among all parallelepipeds $\mathbb{R}^3$ of fixed area $A > 0$, describe (if possible) the ones having maximum volume.

4. Let $F(x,y,z) = z^2 - xy - 1$. Determine the points of the submanifold of $\mathbb{R}^3$ defined by $F = 0$ which are the closest to the origin.